647510 **MSC4MatC401x** **Seat No :________**

M.Sc.(Maths) Semester - 4 (*CBCS*) Examination
May/June-2021 (NEW COURSE)
Linear Algebra (CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1 (A) Answer the following question. **[04]**

(1) Let $B = \{v_1, v_2\}$ be a basis of a vector space V. Let $T: V \to V$ be defined as $T(v_1) = v_1 + v_2, T(v_2) = v_1 - v_2$. Find Characteristic roots of *T*.

Q.1 (B) Answer any two of the following questions. **[10]**

(1) Prove that if *A* is an algebra with unit element over a field *F*, then A is isomorphic to a subalgebra of *A(V)* for some vector space *V* over *F*.

(2) In standard notations prove that if $T \in A(V)$ and if $dim_F V = n$then T can have at most $n$distinct characteristic roots in F.

(3) Give an example of a linear transformation which is right invertible but not invertible.

Q.2 (A) Answer the following question. **[04]**

(1) Prove that $T \in A(V)$ is nilpotent if and only if 0 is the only characteristic root of T.

Q.2 (B) Answer any two of the following questions. **[10]**

(1) Prove that if $T \in A(V)$ has all its characteristic roots in F, then there exists a basis of V with respect to which matrix of T is triangular.

(2) Prove that the set D of all matrices commuting with $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$is isomorphic to the field of complex numbers.

(3) Prove that two nilpotent linear transformations are similar if and only if they have same invariants.

Q.3 (A) Answer the following question. **[04]**

(1) Determine the number of possible Jordan canonical forms of a 6×6 matrix having minimal polynomial $x^2(x-3)^3$.

Q.3 (B) Answer any two of the following questions. **[10]**

(1) Let $V = R^3, IF\ T{:}R^3 \to IR^3$ be defined as $T\big((x,y,z)\big) = (2x,y,z) \forall (x,y,z) \in R^3$. Find two subspaces $V_1$and V_2invariant under Tsuch that $V = V_1 + V_2$.

(2) In standard notations prove that $S, T \in A(V)$ are similar if and only if they have the same elementary divisors.

(3) Prove that the matrix $\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$is nilpotent and find its invariants and Joradan form.

Q.4 (A) Answer the following question. **[04]**

(1) For $A, B \in F_n$ prove that $tr(AB) = tr(BA)$ and hence deduce that $tr(ABA^{-1}) = tr(B)$ if A is invertible.

Q.4 (B) Answer any two of the following questions. **[10]**

(1) Prove that if two rows of a matrix A are equal then $det(A) = 0$.

(2) State and prove necessary and sufficient condition for a linear transformation T on a vector space V to be unitary.

(3) Prove that a Hermitian linear transformation T is nonnegative if and only if all its characteristic roots are nonnegative.

Q.5 (A) Answer the following question. **[04]**

(1) Define nondegenerate bilinear map and check whether the map $f: R^2 \times R^2 \to R$, $f\big((x_1, y_1), (x_2, y_2)\big) = x_1x_2 + y_1y_2\big((x_1, y_1), (x_2, y_2) \in R^2\big)$ is degenerate or not.

Q.5 (B) Answer any two of the following questions. **[10]**

(1) Let V be a finite dimensional vector space over a field F with characteristic 0. Let $f: V \times V \to F$ be a symmetric bilinear form. Show that there exists a basis B of V such that the matrix of f in B is diagonal.

(2) Let V be a finite dimensional vector space over a field F and $f: V \times V \to F$ be a non-degenerate bilinear form. Show that the set $G = \{T \in A(V): T\ preseves\ f\}$ forms a group under composition of operators.

(3) Determine the rank and signature of the quadratic form

$$x^2 + 6y^2 + 2z^2 + 4xy + 10xz + 22yz.$$

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